In a nutshell: The method of successive over-relaxation

Given an iterative problem where $x_{k+1} \leftarrow f(x_k)$, then under certain conditions, we may be able to find a relaxation parameter ω to speed up convergence. Thus, having calculated x_{k+1}, we then update the value with

$$x_{k+1} \leftarrow \omega x_{k+1} + (1 - \omega)x_k$$

In specific cases (e.g., Newton's method), we may calculate Δx_k so that $x_{k+1} \leftarrow x_k + \Delta x_k$, in which case, applying over-relaxation amounts to calculating

$$x_{k+1} \leftarrow x_k + \omega \Delta x_k$$

Now, specific to the Gauss-Seidel method, given a system of n linear equations in n unknowns $A\mathbf{u} = \mathbf{v}$, we will use iteration to approximate a solution to this system of linear equations. We will assume that A is either strictly diagonally dominant or symmetric and positive definite, in which case, we are assured that all the diagonal entries are non-zero.

Parameters:

ω	A relaxation parameter ω to speed up convergence.
$\varepsilon_{\text{step}}$	The maximum step size allowed before we consider the method to have converged.
N	The maximum number of iterations.

1. Define A_{diag} to be the $n \times n$ matrix of the diagonal entries of A and calculate the inverse A_{diag}^{-1} of this matrix, which is that matrix with the reciprocals of each of the diagonal entries of A_{diag}.
2. Define A_{off} to be the $n \times n$ matrix of the off-diagonal entries of A.
3. Let $\mathbf{u}_0 \leftarrow A_{\text{diag}}^{-1}\mathbf{v}$ and $k \leftarrow 0$.
4. If $k > N$, we have iterated N times, so stop and return signalling a failure to converge.
5. Set $\mathbf{u}_{k+1} \leftarrow \mathbf{u}_k$.
6. For i going from 1 to n,
    a. Update the i^{th} entry of $(\mathbf{u}_{k+1})_i \leftarrow (1-\omega)(\mathbf{u}_{k+1})_i + \omega\left(A_{\text{diag}}^{-1}\right)_{i,i}\left(\mathbf{v}_i - (A_{\text{off}})_{i,\cdots}\mathbf{u}_{k+1}\right)$ where $(A_{\text{off}})_{i,\cdots}$ is the i^{th} row of A_{off}.
7. If $\|\mathbf{u}_{k+1} - \mathbf{u}_k\|_2 < \varepsilon_{\text{step}}$, return $\mathbf{u}_{k+1}$.
8. Increment k and return to Step 4.

Note that if A is a sparse matrix (most entries are zero and stored using a sparse-matrix representation), then it is reasonable to calculate $A_{\text{diag}}^{-1}A_{\text{off}}$ first and then replace Step 6*a* by:

6a′. Update the i^{th} entry of $(\mathbf{u}_{k+1})_i \leftarrow (1-\omega)(\mathbf{u}_{k+1})_i + \omega\left(\mathbf{u}_{0,i} - \left(A_{\text{diag}}^{-1}A_{\text{off}}\right)_{i,\cdots}\mathbf{u}_{k+1}\right)$ where $\left(A_{\text{diag}}^{-1}A_{\text{off}}\right)_{i,\cdots}$ is the i^{th} row of $A_{\text{diag}}^{-1}A_{\text{off}}$.